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TEMPERATURE MEASUREMENTS USING A BIREFRINGENT SENSOR

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An optical method for temperature measurements is proposed based on the use of an autocollimation polarimeter and a sensitive birefringent plate. The basic expressions that describe the operation of the system are presented. It is shown that with the use of a quartz plate 1.5 mm thick the device makes it possible to determine values of temperature in the region of −80 to +150°C.

On numerous occasions temperature monitoring must be performed under conditions where the direct connection of the object under investigation and the measuring device is either impossible or difficult to arrange. In these cases optical methods based on pyrometry and use of color change in liquid crystals or thermopaints are of crucial importance [1, 2].

It is well known that the magnitude of birefringence in crystals is strongly dependent on temperature [3–5]. This property makes it possible to design a quick-response remote-indicating temperature sensor. For this purpose a plane-parallel birefringent crystal plate is set in the medium under investigation and illuminated with a polarized light beam. The plate decomposes the incident light into two polarized components and introduces a phase shift between them. The phase shift introduced by the plate can be determined from the change in the polarization parameters of the radiation, and then one can determine the temperature value of the medium. The use precisely of the polarization properties of light makes it possible to decrease substantially the effect of beam attenuation on the measurement errors in the case where the radiation passes through smokey or dusty space. The small dimensions of the sensitive crystal plate provide a rather fast response of the sensor to temperature variations in the medium.

The back-reflection design with the compensation method of registration [6] is convenient for practical implementation of the measuring instrument. In this case the beam passes through the plate twice, which improves the sensitivity of the method. In addition, the following features are advantages of the system: a) the possibility of carrying out remote sensing of the parameters of objects that are difficult to access directly; b) the use of one and the same window for the input and output of radiation when the object is placed in a hermetically sealed chamber.

The measuring device consists of the source of polarized collimated light *I*, polarimetric unit *II*, modulator *III*, and indicator unit *IV* (see Fig. 1). The sensor is indicated as *V*. An He-Ne laser can be used as a radiation source. The elements of the polarimetric unit are as follows: the fixed attachment 1, the first rotating body 2, the first goniometric limb 3, the second rotating body 4, the second goniometric limb 5, the first $\lambda/4$ plate 6, the first polarizer 7, the polarization beam splitter 8, the second polarizer 9, the photodiode 10, and the second $\lambda/4$ plate 11. The modulator should not affect the polarization of the beam, and therefore a disk with slits 12 is used in the system, which is driven by the motor 13. It is better to mount the modulator disk 12 in the beam between the polarimetric unit and the sensor to avoid modulation of the laser radiation scattered from the passing probe beam within the polarimetric unit. The sensor consists of the plane-parallel crystal plate 14 and mirror 15. The indicator unit consists of the narrow-band amplifier 16, amplitude detector 17, and indicator device 18.

The original laser beam with linear polarization with fixed azimuth is incident on the polarimetric unit. Using a system that consists of the first $\lambda/4$ plate, the first polarizer, and the polarization beam splitter, the azimuth

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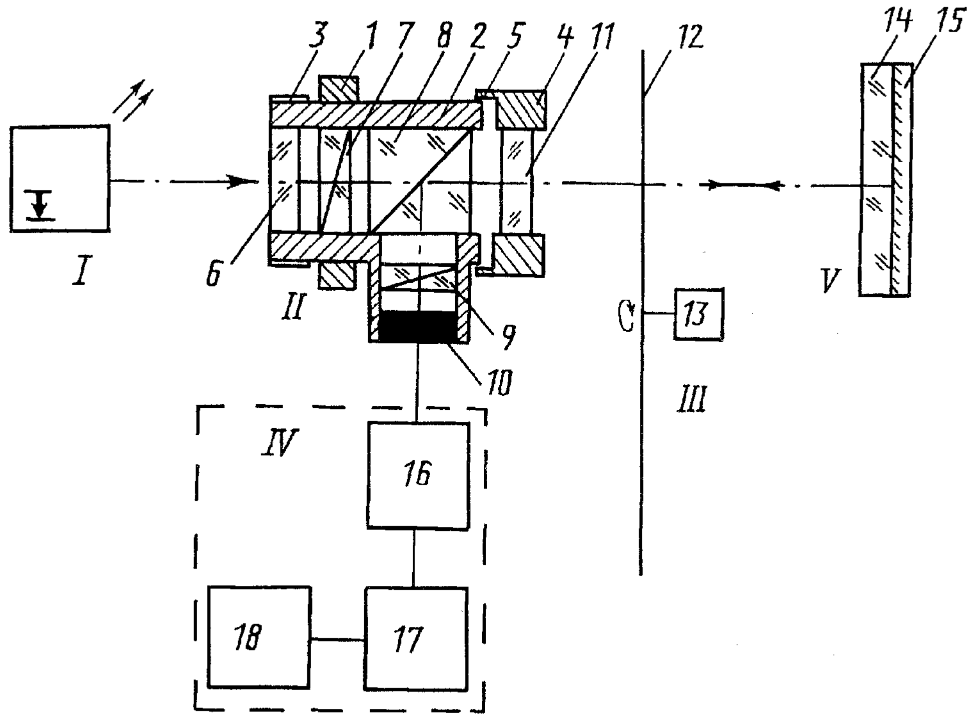

Fig. 1. Block diagram of the temperature meter.

of the linearly polarized light is turned through the desired angle. In this case the output beam intensity is kept constant.

Then the linearly polarized probe beam illuminates the sensitive element, which reflects the radiation backward. If the polarization plane of the probe beam is parallel to the principal planes of the crystal plate then the reflected beam retains its polarization. In the case where the polarization plane of the probe beam does not coincide with the principal planes of the plate the polarization state of the reflected beam is changed. As a consequence, a component perpendicular to the polarization plane of the probe beam appears in the reflected beam. The orthogonal component of the reflected beam is directed to the photodiode 10 by the beam splitter 8. Since a beam splitter with an interference coating is utilized in the unit, the additional polarizer 9 is employed. It makes it possible to eliminate the small portion of the collinear polarization component that remains after reflection from the beam splitter. The presence of an orthogonal component in the radiation incident on the photodiode 10 is determined by the indicator unit.

The intensity of the radiation flux of the orthogonal component Φ' in the reflected beam is described by the following expression [6]:

$$\Phi' = \left\{ \sin\frac{\Delta}{2} \cos 2(\varphi - \vartheta) \sin 2(\varphi - \psi) - \cos\frac{\Delta}{2} \sin 2(\varphi - \vartheta) \right\}^2 R^2 \Phi , \tag{1}$$

where Φ is the radiation flux of the probe beam; R is the amplitude reflection coefficient of the mirror; Δ is the phase difference introduced into the beam by the crystal plate upon double passage of the radiation; φ, ψ, and ϑ are the angular positions of the fast axes of the second $\lambda/4$ plate and sensitive crystal plate, as well as of the principal plane of the polarization beam splitter which specifies the polarization plane of the probe beam with respect to the zero direction of the device.

When the principal planes of the polarization beam splitter and the second $\lambda/4$ plate coincide (i.e., $\varphi = \vartheta$) Eq. (1) can be put in the following form:

$$\Phi' = \sin^2\frac{\Delta}{2} \sin^2 2(\psi - \vartheta) R^2 \Phi . \tag{2}$$

In this case compensation of the radiation flux Φ' is achieved by simultaneous rotation of the polarization beam splitter and the second $\lambda/4$ plate. In the position found, the orientation of the principal planes of the sensitive crystal plate corresponds to the orientation of the principal planes of the polarization beam splitter.

Furthermore, we install the second $\lambda/4$ plate in such a way that its fast axis makes an angle of 45° with the fast axis of the sensitive crystal plate, i.e., $\varphi - \psi = 45^{\circ}$. In this case

$$\Phi' = \sin^2 \left[\frac{\Delta}{2} - 2(\varphi - \vartheta) \right] R^2 \Phi . \tag{3}$$

By rotating the polarization beam splitter with the second $\lambda/4$ plate fixed, one can attain compensation of the flux Φ 'of the orthogonal component. The difference in the azimuths of the beam splitter and the second $\lambda/4$ plate, the latter being in the position found, makes it possible to determine the value of the phase shift of the sensitive crystal plate:

$$\Delta = 4(\varphi - \vartheta) + 360^{\circ} m , \tag{4}$$

where m is an integer.

In the general case the temperature affects both the phase shift $\Delta = \Delta(t)$, which is directly proportional to the magnitude of the birefringence and to the thickness of the plate, and the position of the phase axis $\psi = \psi(t)$ [7]. However, the latter property can be observed only for crystals with low symmetry, the magnitude of this effect being insignificant. Polarization eigencomponents of radiation are known to be temperature-independent in uniaxial crystals. In what follows we will assume that no rotation of the fast axis of the crystal plate is induced by the temperature deformation of its holder as well.

Let us use a sensitive crystal quartz plate cut parallel to the optical axis to illustrate the operation of the device. In the temperature region of -200°C up to the phase transition point at $+573^{\circ}$C quartz possesses good optical quality and stable characteristics [8]. The phase shift $\Delta(t)$ introduced by the plate at normal incidence and double passage of the beam is defined by the expression

$$\Delta(t) = \frac{720^{\circ}}{\lambda} l(t) \left[n_e(\lambda, t) - n_0(\lambda, t) \right] , \tag{5}$$

where λ is the wavelength of the radiation in vacuum; $l(t)$ is the thickness of the plate; $n_e(\lambda, t)$ and $n_0(\lambda, t)$ are the refractive indices for the extraordinary and ordinary rays, respectively.

The temperature change in the thickness of the quartz plate in the temperature region of -200 to $+500^{\circ}$C is approximated adequately by the quadratic function [3]

$$l(t) = l_0 (1 + A_{\perp} t + B_{\perp} t^2) , \tag{6}$$

where l_0 is the thickness of the plate at $t = 0^{\circ}$C; $A_{\perp}$and $B_{\perp}$ are the linear and quadratic coefficients of the thermal expansion of quartz in the direction perpendicular to the crystallophysical axis with $A_{\perp} = 13.24 \cdot 10^{-6}$ deg^{-1}, $B_{\perp} = 1.34 \cdot 10^{-8}$ deg^{-2}.

The relationship between the magnitude of the birefringence and the value of the temperature is usually given by a linear approximation [3, 4]:

$$n_e(\lambda, t) - n_0(\lambda, t) = n_e(\lambda) - n_0(\lambda) + \left[\frac{dn_e(\lambda, t)}{dt} - \frac{dn_0(\lambda, t)}{dt} \right] t , \tag{7}$$

where $n_e(\lambda) - n_0(\lambda)$ is the magnitude of the birefringence at $t = 0^{\circ}$C. In the case where an He-Ne laser with $\lambda = 632.8$ nm is used as the excitation source we obtain

$$n_e(\lambda) - n_0(\lambda) = 9.1 \cdot 10^{-3} , \quad \frac{dn_e(\lambda, t)}{dt} -$$

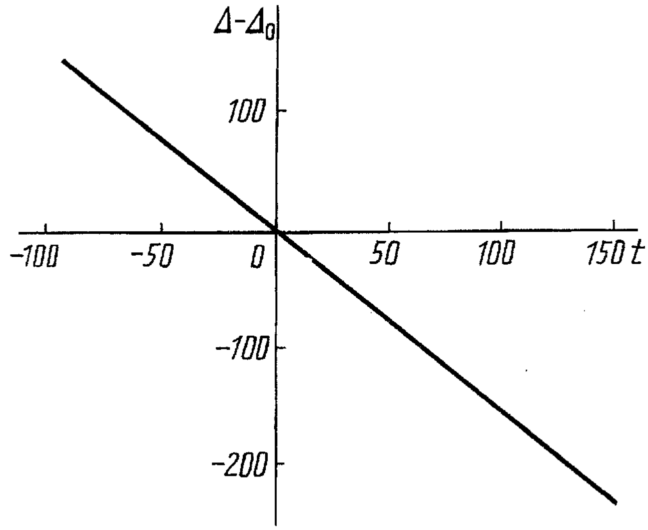

Fig. 2. Dependence of the increase in the phase shift (deg) on the temperature (^{o}C) for a quartz plate 1.5 mm thick.

$$-\frac{dn_0(\lambda,\ t)}{dt} = -\ 1.04 \cdot 10^{-6}\quad \text{deg}^{-1}.$$

Substituting expressions (6) and (7) into expression (5), one can obtain an equation for the temperature dependence of the increase in the phase shift of the plate:

$$\Delta(t) - \Delta_0 = [\mu + (1 + \mu t)(A_\perp + B_\perp t)\, \Delta_0 t\,. \tag{8}$$

Here

$$\Delta_0 = \frac{720^\circ}{\lambda}\, l_0\, [n_e(\lambda) - n_0(\lambda)]\,, \quad \mu = \frac{\dfrac{dn_e(\lambda,\ t)}{dt} - \dfrac{dn_0(\lambda,\ t)}{dt}}{n_e(\lambda) - n_0(\lambda)}\,. \tag{9}$$

Taking into account that $(A_\perp \mu + B_\perp)t/(A_\perp + \mu) << 1$ Eq. (8) can be simplified as follows:

$$\Delta(t) - \Delta_0 = (\mu + A_\perp)\, \Delta_0 t\,. \tag{10}$$

Figure 2 shows the temperature dependence of $\Delta(t) - \Delta_0$ for a quartz plate 1.5 mm thick, with the results obtained using Eqs. (8) and (10) being practically coincident. As is seen from Fig. 2, the phase shift of the plate changes linearly with temperature within the limits of one period from 127 to -233^{o} in the temperature range of -80 to $+150^{o}$C.

Equation (1) makes it possiible to estimate the temperature region $t_{max} - t_{min}$ within which the phase shift $\Delta(t)$ changes within the limits of one period:

$$t_{max} - t_{min} = \left| \frac{360^\circ}{(\mu + A_\perp)\, \Delta_0} \right|. \tag{11}$$

We substitute the values of the parameters according to (9) into this expression:

$$t_{max} - t_{min} = \frac{\lambda}{\left| 2l_0 \left[\left(\dfrac{dn_e}{dt} - \dfrac{dn_0}{dt} \right) + A_\perp (n_e - n_0) \right] \right|}\,. \tag{12}$$

In the case where a quartz plate with $l_0 = 1.5$ mm is used the value of $t_{max} - t_{min}$ equals 230^{o}C.

The measuring device can also determine the value of the phase shift Δ with an accuracy of one period (see Eq. (4)). Therefore, having determined the azimuth difference $\varphi - \vartheta$ at which the flux of the orthogonal polarization component is compensated, one can unambiguously determine the temperature of the quartz plate under consideration in the region of -80 to $+150^\circ$C. Using the use of the expression (10), we obtain

$$t = \frac{\Delta(t) - \Delta_0}{(\mu + A_\perp)\,\Delta_0}. \tag{13}$$

In principle, if the parameters Δ_0 and μ (see Eq. (9)) were calculated previously, then just one measurement of the value of Δ is required (see Eq. (4)) to obtain the desired temperature value according to formula (3). However, in this case $\Delta(t) - \Delta_0 << \Delta_0$. Therefore, even a minor relative error in the parameter Δ_0 caused by the inaccuracy in fabrication or installation of the sensitive plate can lead to a substantial error in the temperature measurement.

This can be avoided if one first determines the position of compensation at a known temperature $\tilde{t}$. Taking into account that $\psi(t)$ = const and, correspondingly, $\varphi(t) = \psi(t) + 45^\circ$ = const, one can derive from Eq. (4) the system of equations

$$\Delta(t) = 4(\varphi - \vartheta(t)) + 360^\circ m, \quad \Delta(\tilde{t}) = 4(\varphi - \vartheta(\tilde{t})) + 360^\circ m, \tag{14}$$

where m is an integer.

On the other hand, dependence (13) can be written in the form

$$t = \tilde{t} + \frac{\Delta(t) - \Delta(\tilde{t})}{(\mu + \Delta_\perp)\,\Delta_0}. \tag{15}$$

Having determined the phase shift difference $\Delta(t) - \Delta(\tilde{t})$ using system (14), we finally arrive at

$$t = \tilde{t} + \kappa\,[\vartheta(t) - \vartheta(\tilde{t})], \tag{16}$$

where $\kappa = -4/(\mu + A_\perp)\Delta_0$. In particular, for the 1.5-mm-thick quartz plate under consideration $\kappa = 2.54^\circ$C/deg.

Thus, in order to find the temperature of the plate one needs to find the value of the difference in azimuths of the polarization beam splitter $\vartheta(t) - \vartheta(\tilde{t})$ for the known temperature and the temperature being measured for compensation of the orthogonal polarization component in the reflected beam. Thereafter, one can easily obtain the desired quantity using Eq. (16).

In a particular device one needs to find the position $\vartheta(\tilde{t})$ only once and take it thereafter as a reference point. The scale of the first goniometric limb is conveniently calibrated in $^\circ$C in the corresponding scale. This makes it possible to obtain directly the temperature values in the course of measurements.

Equation (16) makes it possible to determine the temperature for changes in the phase shift of the plate within the limits of one period. If the values of the phase shifts $\Delta(t)$ and $\Delta(\tilde{t})$ correspond to different periods, which occurs when $|t - \tilde{t}| > t_{max} - t_{min}$ (see Eq. (12)), then the following expression should be employed:

$$t = \tilde{t} + \kappa\,[\vartheta(t) - \vartheta(\tilde{t})] - 90^\circ \kappa j, \tag{17}$$

where j is the difference in the numbers of the periods $\Delta(t)$ and $\Delta(\tilde{t})$.

Inasmuch as rotation of the beam splitter by 90° corresponds to one period of change in the phase shift Δ (see (14)), the range of temperature measurements $t_{max} - t_{min}$ is related to the sensitivity κ of the detector by the dependence

$$t_{max} - t_{min} = 90^\circ\,|\kappa|. \tag{18}$$

This relationship is easily verified by comparing (12) with (16).

As is seen from the Eq. (16) the error in the determination of the temperature is proportional to the error in the measurement of the azimuth ϑ. To improve the sensitivity of the detector one should decrease the value of

κ. This can be achieved by using a plate of greater thickness made of a material with a steeper temperature dependence of the birefringence and a higher thermal expansion coefficient. At the same time one should take into account that an increase in the sensitivity of the detector is accompanied by a narrowing of the measurable temperature interval (see (18)).

Inasmuch as the parameter $n_e - n_0$ is canceled from the product $\mu\Delta_0$ (see (9)) and $\mu >> A_\perp$ in this case, the magnitude of the birefringence will not affect the sensitivity of the detector noticeably. However, a large value of $n_e - n_0$ results in a considerable uncertainty in the inclination of the plate. Let us estimate the value of the permissible inclination of the plate in temperature measurements.

Let us consider an example of a sensitive plate produced from an optically uniaxial crystal in such a way that its optical axis is parallel to the surface of the plate. If such a plate is turned about an axis that lies in the plane of the plate and makes an angle χ with the optical axis, the phase shift will be defined by the following formula [9]:

$$\Delta = \Delta_0 \frac{\sqrt{n_0^2 n_e^2 - (n_e^2 - (n_e^2 - n_0^2)\cos^2\chi)\sin^2\varepsilon} - n_0\sqrt{n_0^2 - \sin^2\varepsilon}}{n_0 (n_e - n_0)}, \tag{19}$$

where ε is the angle of incidence of the beam on the plate.

Normally, the value of the birefringence $n_e - n_0$ is two orders of magnitude less than the absolute values of the refraction indices n_e and n_0. Therefore, the expression (19) can be written in the approximate form

$$\Delta = \Delta_0 \sqrt{\left(1 - \left(\frac{\sin\varepsilon}{\tilde{n}}\right)^2\right)} \left[1 + \frac{\cos^2\chi}{1 - \left(\frac{\sin\varepsilon}{\tilde{n}}\right)^2} \left(\frac{\sin\varepsilon}{\tilde{n}}\right)^2\right], \tag{20}$$

where $\tilde{n} \equiv (n_e + n_0)/2$. The remainder in the series expansion has a value of the order of $[(n_e - n_0)\cos^2\chi\sin^2\varepsilon/(2\tilde{n}^5)]\Delta_0$. In particular, at $n_e \approx n_0 \approx 1.55$, $n_e - n_0 = 0.009$, $\varepsilon \le 2^\circ$ the error of the approximation is not greater than $10^{-9}\sin^2\chi\Delta_0$, which is a negligibly small quantity. Furthermore, the angle of incidence of the beam is close to the normal one and correspondingly the parameter $(\sin\varepsilon/\tilde{n})^2$ is also small. We represent (19) in the form of an expansion in this parameter. With accuracy up to the remainder $[(1 + 4\cos^2\chi)\sin^4\varepsilon/(8\tilde{n}^4)]\Delta_0$ we obtain

$$\delta\Delta \equiv \Delta - \Delta_0 = \frac{\cos 2\chi}{2}\left(\frac{\sin\varepsilon}{\tilde{n}}\right)^2 \Delta_0 . \tag{21}$$

If $\tilde{n} = 1.55$ and $\varepsilon \le 2^\circ$ then the value of the remainder in (20) is comprises less than $2\cdot 10^{-7}\,\Delta_0$ and it can be neglected.

The magnitude of the error δt of the temperature measurement caused by the inclination of the plate can be found by differentiating Eq. (10) and then substituting the value of $\delta\Delta$ expressed according to (21) into the expression found:

$$\delta t = \frac{\delta\Delta}{(\mu + A_\perp)\Delta_0} = \frac{\cos 2\chi}{2(\mu + A_\perp)}\left(\frac{\sin\varepsilon}{\tilde{n}}\right)^2 . \tag{22}$$

As is clear from this expression, the measurement error caused by the inclination of the plate does not depend on its thickness and is determined solely by the properties of the material. The maximum error is introduced at an inclination of the plate in the plane that is perpendicular ($\chi = 0^\circ$) or parallel ($\chi = 90^\circ$) to the optical axis. We note that in these two cases the error δt has opposite signs.

In many types of crystals the parameter μ is substantially greater than the quantity $A_\perp$. For example, in quartz $A_\perp \approx 0.1\mu$ at $\lambda = 633$ nm. In this case

$$\delta t = \frac{n_e - n_0}{2\left(\frac{dn_e}{dt} - \frac{dn_0}{dt}\right)} \cos 2\chi \left(\frac{\sin \varepsilon}{\tilde{n}}\right)^2. \tag{23}$$

This relation makes it possible to find the restriction that should be imposed on the admissible angle of inclination of the plate during measurements. If the magnitude of the inclination is small, then the maximum value of the inclination angle of the plate expressed in degrees is as follows:

$$\varepsilon_{\max} = \frac{180^\circ}{\pi} \tilde{n} \sqrt{\left(\left|\frac{2\left(\frac{dn_e}{dt} - \frac{dn_0}{dt}\right)\delta t}{n_e - n_0}\right|\right)}. \tag{24}$$

As follows from (24), the admissible inclination angle of quartz plates that introduces an error δt of no more than 0.01°C into the result of a measurement of temperature should not exceed 0.1°. Such an inclination value causes a deflection of the reflected beam of ±5 mm for a distance of 1 m from the plate, which can easily be observed visually. In the case where the inclination exceeds the admissible value the measured temperature value should be corrected according to (23).

Equation (23) makes it possible to suggest two ways of reducing the error introduced by inclinations of the plate. First, it is recommended that the plates be made from crystals with a steep temperature dependence of the birefringence. Therefore, it may be beneficial to choose the spectral range in the vicinity of the absorption edge of the plate and use materials with a phase transition, e.g., liquid crystals [10]. Second, it is desirable to choose a material with a small absolute value of birefringence. This condition is met by uniaxial crystals with an isotropic point [11] and biaxial crystals with temperature inversion of the optical axes [12].

In conclusion, it should be pointed out that the calculations performed have an estimative nature. Piezooptical phenomena that take place as a result of occurrence of internal strains were not accounted for, and nor were multiple reflection of light within the plate, gyrotropic properties of quartz, etc. The factors just mentioned do not affect the general picture of the results substantially, and, if necessary, they can be accounted for in each individual case or eliminated by an appropriate choice of the design and material of the plate. The formulas derived in the present work can serve as a basis for preliminary analysis and optimization of devices that utilize sensitive plates of other materials and in other temperature ranges.

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